Math 220, Discrete Mathematics, Spring 2017
Midterm 2 Practice Test

Instructions:

- **Please read each question carefully.**

- **No calculators, notes, books, or outside help of any kind are allowed to be used on this exam. Please turn cell phones off!**

- **Show all of your work and explain your answers clearly. In order to receive full credit your work must be complete, clear, and logical.**

- **Please cross out or fully erase any work that you do not want graded.**

1. Define a relation $\mathcal{R}$ on $\mathbb{R}^2$ by $(a, b)\mathcal{R}(c, d)$ iff $a \leq c$.

   (a) Show that $\mathcal{R}$ is transitive.

   (b) Show that $\mathcal{R}$ is not an equivalence relation.

   (c) Show that $\mathcal{R}$ is not a partial order relation.

2. Give an example of a partially ordered set that is not totally ordered.

3. State and prove the divisibility test for:

   (a) 11

   (b) 18

4. Define $f : \mathbb{R}^2 \to \mathbb{R}$ by $f((a, b)) = a - b$.

   (a) Determine whether f is onto.

   (b) Find $f^{-1}(\{0\})$.

   (c) Determine whether f is invertible.

5. Prove the following statement or give a counterexample: Let $a, b, c \in \mathbb{N}$. If $\gcd(a, b) \neq 1$ and $a | (b + c)$ then $\gcd(a, c) \neq 1$.

6. Compute the following:

   (a) The number of ways of ordering 3 *different* one topping pizzas, given that there is a choice of 10 possible toppings.

   (b) The number of different license plates if each contains 1 number followed by two letters followed by 3 numbers.

   (c) The number of positive divisors of $2^8 3^2 5^7 11^3$.

   (d) The number of cards to be dealt from a standard deck to guarantee 5 cards of the same suit.